DOCTORAL PROJECT PROPOSAL
DOCTORAL SCHOOL OF EXACT AND NATURAL SCIENCES
NICOLAUS COPERNICUS UNIVERSITY IN TORUŃ
Contest 005, May 2023

Project title (in English)

Global bifurcations of periodic solutions of autonomous Hamiltonian systems

Project title (in Polish)

Globalne bifurkacje rozwiązań okresowych autonomicznych układów hamiltonowskich

Project submitter: prof. Sławomir Rybicki, rybicki@mat.umk.pl, 669980850

Suggested supervisors and mentors

Main supervisor: prof. Sławomir Rybicki, rybicki@mat.umk.pl, 669980850, Faculty of Mathematics and Computer Science, Nicolaus Copernicus University in Toruń, discipline: Mathematics

Auxiliary supervisor: dr Anna Gołębiewska, aniar@mat.umk.pl, 503 480 157,Faculty of Mathematics and Computer Science, Nicolaus Copernicus University in Toruń, discipline: Mathematics

1. **PHD PROJECT DESCRIPTION (4000 characters max., including the aims and work plan)**
   Project title:

   Global bifurcations of periodic solutions of autonomous Hamiltonian systems

1.1. **Project goals:** The main aim of the project is to characterize the sets of solutions of some Hamiltonian systems derived from models of physical phenomena, with particular emphasis on celestial mechanics models. The first type of systems that will be studieded is the Hénon-Heiles system and its various generalizations, which are used to model many physical phenomena. The second type of considered problems are Hamiltonian systems modeling the motion of Saturn's rings. For the above-mentioned systems, it is planned to investigate the global bifurcation phenomenon of non-trivial periodic solutions, and in particular to study the existence of unbounded closed connected sets of such solutions.

1.2 **Outline:**

The Hamiltonian of the Hénon-Heiles system is a third-degree polynomial, depending on three parameters, so it is relatively simple. However, its dynamics is so complicated that it has become a challenge for many researchers. To our knowledge, there are no results regarding global bifurcation of periodic solutions of this type of systems in the scientific literature. The only known bifurcation results were obtained with the use of Lyapunov's Center Theorem, and Theorems of Reeb and Weinstein, therefore they concern local bifurcations, not global. In the project it is planned to prove global results with the use of the approach initiated by Paul Rabinowitz in the years 70s of the last century. The idea of this approach is to prove the existence of periodic solutions of Hamiltonian systems as critical points of functionals defined on appropriately chosen Hilbert spaces. It is known, that if the considered system is autonomous, then the corresponding functional is S^1-invariant. Continuing this approach, using contemporary methods of invariant non-linear analysis, will allow to obtain new results concerning continua of non-stationary periodic solutions. Such methods, developed intensively over the last three decades, are the degree for S^1-equivariant gradient mappings and the equivariant Conley index. To compute these invariants we plan to use the theory of normal forms of linear Hamiltonian systems.

1.3 **1.3. Work plan:**

1.3(a) Studying the theory of normal forms of linear Hamiltonian systems.

1.3(b) Investigation of the properties of S^1-invariant functionals whose S^1-critical orbits are periodic solutions of autonomous Hamiltonian systems.

1.3(c) Characterization of trivial solutions, from which global bifurcation of non-stationary solutions of the periodic Hamiltonian system can occur (necessary condition).

1.3(d) Computations of Brouwer's indices of isolated critical points of the Hamiltonian and of bifurcation indices in terms of degree for S^1-equivariant gradient maps (sufficient condition). The classification of normal forms of linear Hamiltonian systems will be used here.

1.3(e) Application of the global bifurcation theorem for periodic solutions of autonomous Hamiltonian systems to study global bifurcations of non-stationary periodic solutions of certain Hamiltonian systems of celestial mechanics and Newtonian systems with Coriolis forces, including the generalized Hénon-Heiles system and the Hamiltonian system modeling the motion of Saturn's rings.

1.3(f) Comparison of the obtained results with those existing in the literature.

1.4 **Literature** (max. 10 listed, as a suggestion for a PhD candidate)

(1) A. Gołębiewska, E. Pérez-Chavela, S. Rybicki, A. Ureña, Bifurcation of closed orbits from equilibria of Newtonian systems with Coriolis forces, Journal of Differential Equations 338 (2022), 441-473
(2) K.R. Meyer. G.R. Hall & D. Offin, *Introduction to Hamiltonian dynamical systems nd the N-body problem,* Applied Mathematical Sciences **90**, Springer, 2009,
(3) M. Izydorek, *Equivariant Conley index in Hilbert spaces and applications to strongly indefinite problems*, Nonlinear Analysis TMA **51(1)** (2002), 33-66,
(4) E. N. Dancer & S. Rybicki, *A note on periodic solutions of autonomous Hamiltonian systems emanating from degenerate stationary solutions*, Differential and Integral Equations **12(2)** (1999), 147-160,
(5) M. Struwe, *Variational methods. Applications to nonlinear partial differential equations and Hamiltonian systems*, Springer, 1996,
(6) K. Gęba, *Degree for gradient equivariant maps and equivariant Conley index*, Topological Nonlinear Analysis, Degree, Singularity and Variations, (Ed.) M. Matzeu i A. Vignoli, PNDLE **27**, Birkhäuser, (1997), 247-272,
(7) A. Gołębiewska & S. Rybicki, *Global bifurcations of critical orbits of G-invariant strongly indefinite functionals*, Nonlinear Analysis TMA **74(5)** (2011), 1823-1834,
(8) Shui-Nee Chow, Ch. Li & D. Wang, *Normal Forms and Bifurcations of Planar Vector Fields,* Cambridge University Press 1994,
(9) J. Mawhin & M. Willem, *Critical point theory and Hamiltonian systems*, Springer-Verlag, Berlin Heidelberg New York, 1989,
(10) P. H. Rabinowitz, *Minimax methods in critical point theory with applications to differential equations*, Regional Conference Series in Mathematics **65**, AMS, 1985.

1.5 **Required initial knowledge and skills of the PhD candidate:** The candidate should have the ability to think independently, analytically, willingness to self-study and mathematical inquisitiveness. They should also have basic knowledge and skills in the field of mathematical analysis, functional analysis, linear algebra, topology, topological nonlinear analysis and ordinary differential equations.

1.6 **Expected development of the PhD candidate's knowledge and skills:** While implementing the research project, the candidate should conduct research at a high level, acquire the skills to present scientific achievements at a professional level, gain advanced knowledge of critical points theory, invariant topological nonlinear analysis, and theory of Hamiltonian and Newtonian systems.