1. PHD PROJECT DESCRIPTION (4000 characters max., including the aims and work plan)

Project title: Variational, topological and geometrical methods in applications to partial differential equations

1.1. Project goals

The aim of the project is to conduct a quantitative and qualitative analysis of partial differential equations with variational structure originating from mathematical physics. In particular, we will consider equations derived from nonlinear Schrödinger equations, Born-Infeld theory, Einstein-Lichnerowicz scalar field equations, and the Maxwell equations. We expect to obtain abstract results that can be applied to particular equations, introduce new and/or extend existing methods for solving equations (variational, topological, and geometric), as well as creatively combine them.

1.2. Outline

In the project, we mainly employ variational and topological methods, supplemented by geometric methods (derived from differential geometry). In particular, we plan to utilize and extend techniques such as: the mountain pass theorem, the linking geometry, Nehari manifold and Nehari-Pankov manifold, Ekeland variational principle, Pohozaev manifold. A significant role will play as well: theory of differential manifolds (including Riemannian manifolds), Sobolev and Besov space theory, spectral theory of differential operators on manifolds, Lusternik-Schnirelmann category theory, Krasnosel'skii genus, Leray-Schauder topological degree. As part of the project, we will establish theorems concerning existence, multiplicity, and properties of solutions.

1.3. Work plan

The general work plan includes: abstract formulation of the considered problems, proposing assumptions based on physical applications, applying existing tools to the abstract problem, constructing new tools and methods, solving the abstract problem, and interpreting the obtained results in the context of mathematical physics.
In particular, we plan to investigate the following problems:
1) Einstein-Lichnerowicz equations on compact manifolds,

2) Einstein-Lichnerowicz equations on noncompact manifolds,
3) Multiplicity of solutions to Einstein-Lichnerowicz equations,
4) Normalized problems on manifolds,
5) Nonlinear Schrödinger equation with critical Hardy potential,
6) Related Schrödinger and Helmholtz equations on manifolds.

1.4. **Literature** *(max. 10 listed, as a suggestion for a PhD candidate)*

M. Willem: *Minimax theorems*, Birkhäuser 1997
M. Struwe: *Variational methods*, Springer-Verlag 2008
M. Badiale, E. Serra: *Semilinear Elliptic Equations for Beginners*, Springer-Verlag 2011
L.C. Evans: *Równania różniczkowe cząstkowe*, PWN 2002
D. Gilbarg, N.S. Trudinger: *Elliptic partial differential equations of second order*, Springer-Verlag 1983
W. Rudin: *Analiza rzeczywista i zespolona*, PWN 2009
W. Rudin: *Analiza funkcjonalna*, PWN 2012
G. Leoni: *A First Course in Sobolev Spaces*, AMS 2009

1.5. Required initial knowledge and skills of the PhD candidate

A very good understanding of mathematical analysis and functional analysis as well as the theory of partial differential equations is required. Familiarity with analysis on manifolds is also desirable.

1.6. Expected development of the PhD candidate's knowledge and skills

Conducting high-level scientific research, acquiring skills to present scientific achievements at a professional level, gaining advanced knowledge of variational, topological, and geometric methods in the analysis of partial differential equations.